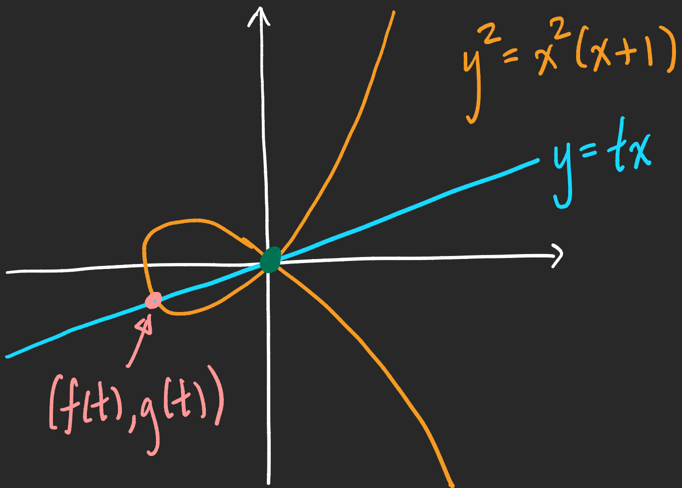


$$\begin{cases} y^2 = x^2(x+1) \\ y = tx \end{cases}$$

$$t^2x^2 = x^2(x+1)$$

$$x^2(t^2 - x - 1) = 0$$

This means $x = 0$ or $x = t^2 - 1$

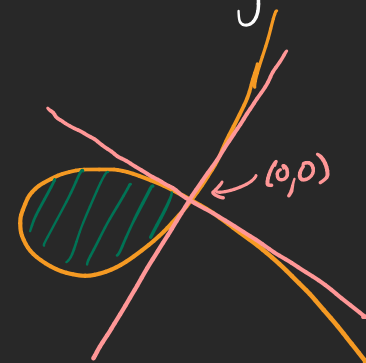
$\Rightarrow y = 0$ gives the origin

$\Rightarrow y = t(t^2 - 1)$

$$f(t) = t^2 - 1$$

$$g(t) = t^3 - t$$

Some questions that one might ask:



1) What are the slopes of these tangents @ (0,0)?

2) What's the area of ▨ enclosed by the "loop" in the curve?

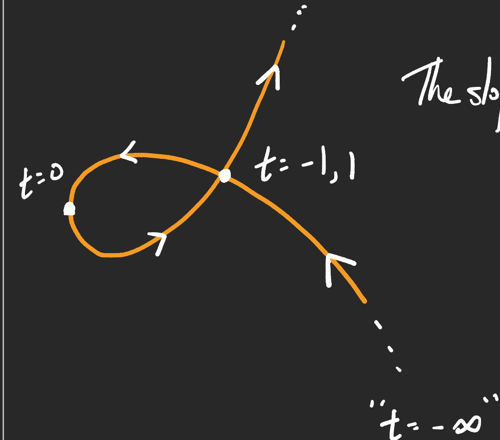
1) Recall $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$ (provided $\frac{dx}{dt} \neq 0$)

$$= \frac{3t^2 - 1}{2t}$$

To find the t values: solve

$$\begin{cases} t^2 - 1 = 0 \\ t(t^2-1) = 0 \end{cases}$$

The only solutions are $t = -1, 1$.



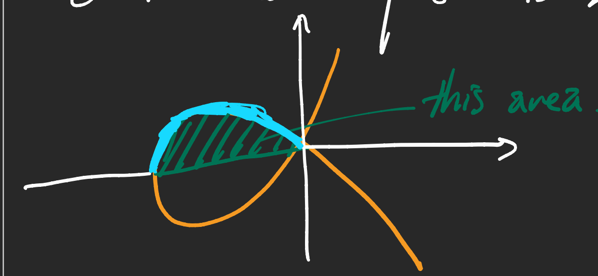
The slopes are thus

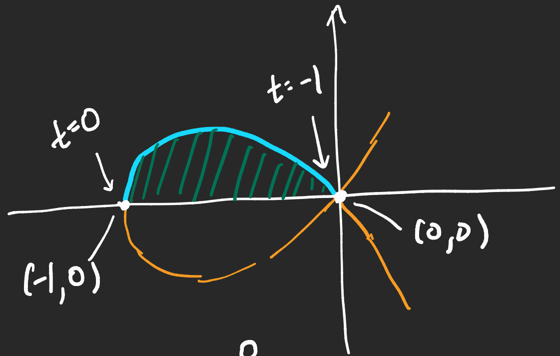
$$\frac{3(-1)^2 - 1}{2(-1)} = -1$$

$$\frac{3(1)^2 - 1}{2(1)} = 1$$

2) Note that $y^2 = x^2(x+1)$ has vertical symmetry since replacing y with $-y$ does not change the equation.

So the area in question is 2 times





Area $= \int_{-1}^{0} y\,dx$ (from single-var calculus.)

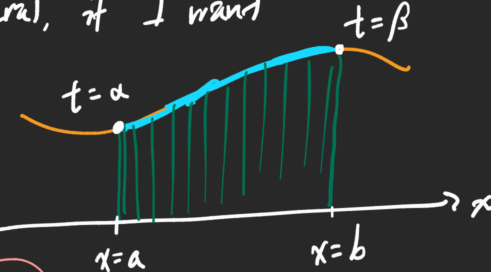
Since $x=f(t)$, $dx = \frac{dx}{dt}\,dt$

$= f'(t)\,dt$

$= 2t\,dt$

$$= \int_{0}^{-1} g(t)f'(t)\,dt = \int_{0}^{-1} (t^3-t)\,2t\,dt$$

In general, if I want



(a<b)

$$= \int_{\alpha}^{\beta} g(t)f'(t)\,dt$$

even if β is smaller!

$$= \int_{0}^{-1} (2t^4-2t^2)\,dt$$

$$= \left(\frac{2}{5}t^5 - \frac{2}{3}t^3\right)\Big|_{t=0}^{-1} = -\frac{2}{5}+\frac{2}{3}$$

$$= \frac{4}{15}$$

So the final answer is $2 \cdot \frac{4}{15} = \boxed{\frac{8}{15}}$.